### Chapter 7

**Fixed-Income Securities**

The next two chapters provide an overview of ﬁxed-income markets, securities, and their derivatives. Originally, **ﬁxed-income securities** referred to bonds that promise to make ﬁxed coupon payments. Over time, this narrow deﬁnition has evolved to include any security that obligates the borrower to make speciﬁc payments to the bondholder on speciﬁed dates. Thus, a **bond** is a security that is issued in connection with a borrowing arrangement. In exchange for receiving cash, the borrower becomes obligated to make a series of payments to the bondholder.

Fixed-income derivatives are instruments whose value derives from some bond price, interest rate, or other bond market variable. Due to their complexity, these instruments are analyzed in the next chapter.

Section 7.1 provides an overview of the different segments of the bond market. Section 7.2 then introduces the various types of ﬁxed-income securities. Section 7.3 reviews the basic tools for analyzing ﬁxed-income securities, including the determi- nation of cash ﬂows, the measurement of duration, and the term structure of inter- est rates and forward rates. Because of their importance, mortgage-backed securities (MBSs) are analyzed separately in Section 7.4. The section also discusses collateralized mortgage obligations (CMOs), which illustrate the creativity of ﬁnancial engineering.

###### Overview of Debt Markets

Table 7-1 breaks down the world debt securities market, which was worth $38 trillion at the end of 2001. This includes the **bond markets**, deﬁned as ﬁxed-income securities with remaining maturities beyond one year, and the shorter-term **money markets**, with maturities below one year. The table includes all publicly tradable debt securities sorted by country of issuer and issuer type as of December 2001.

To help sort the various categories of the bond markets, Table 7-2 provides a broad classiﬁcation of bonds by borrower and currency type. Bonds issued by resident entities and denominated in the domestic currency are called **domestic bonds**. In

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**TABLE 7-1 Global Debt Securities Markets - 2001 (Billions of U.S. dollars)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Country of  Issuer | Domestic | Public | Of which  Financials | Corporates | Int’l | Total |
| United States | 15,655 | 8,703 | 4,517 | 2,434 | 2,395 | 18,049 |
| Japan | 5,820 | 4,576 | 570 | 674 | 96 | 5,915 |
| Germany | 1,475 | 686 | 752 | 36 | 643 | 2,117 |
| Italy | 1,362 | 963 | 330 | 70 | 176 | 1,537 |
| France | 1,050 | 642 | 289 | 119 | 402 | 1,452 |
| United Kingdom | 925 | 407 | 292 | 227 | 757 | 1,682 |
| Canada | 571 | 406 | 92 | 73 | 221 | 792 |
| Spain | 364 | 266 | 55 | 43 | 72 | 436 |
| Belgium | 315 | 222 | 75 | 18 | 54 | 369 |
| Brazil | 316 | 261 | 52 | 3 | 60 | 375 |
| Korea (South) | 305 | 79 | 108 | 118 | 44 | 350 |
| Denmark | 229 | 73 | 144 | 13 | 34 | 263 |
| Sweden | 166 | 85 | 60 | 21 | 89 | 255 |
| Netherlands | 360 | 159 | 151 | 51 | 569 | 930 |
| Australia | 183 | 66 | 68 | 50 | 138 | 321 |
| China | 407 | 291 | 106 | 10 | 13 | 420 |
| Switzerland | 161 | 56 | 82 | 23 | 16 | 177 |
| Austria | 154 | 92 | 59 | 3 | 105 | 259 |
| India | 132 | 131 | 0 | 2 | 4 | 137 |
| Subtotal | 29,950 | 18,161 | 7,801 | 3,988 | 5,887 | 35,837 |
| Others | 602 | 703 | 136 | 125 | 1,624 | 2,226 |
| Total | 30,552 | 18,864 | 7,936 | 4,113 | 7,511 | 38,063 |
| Of which, |  |  |  |  |  |  |
| Eurozone | 5,080 | 3,029 | 1,711 | 340 | 2,020 | 7,100 |

Source: Bank for International Settlements

contrast, **foreign bonds** are those ﬂoated by a foreign issuer in the domestic currency and subject to domestic country regulations (e.g., by the government of Sweden in dollars in the United States). **Eurobonds** are mainly placed outside the country of the currency in which they are denominated and are sold by an international syndicate of ﬁnancial institutions (e.g., a dollar-denominated bond issued by IBM and marketed in London). These should not be confused with Euro-denominated bonds. Foreign bonds and Eurobonds constitute the **international bond market**. **Global bonds** are placed at the same time in the Eurobond and one or more domestic markets with securities fungible between these markets.

**TABLE 7-2 Classiﬁcation of Bond Markets**

|  |  |  |
| --- | --- | --- |
|  | By resident | By non-resident |
| In domestic  currency | Domestic  Bond | Foreign  Bond |
| In foreign  currency | Eurobond | Eurobond |

Coupon payment frequencies can differ across markets. For instance, domestic dollar bonds pay interest semiannually. In contrast, Eurobonds pay interest annually only. Because investors are spread all over the world, less frequent coupons lower payment costs.

Going back to Table 7-1, we see that U.S. entities have issued a total of $15,665 billion in domestic bonds and $2,395 billion in international bonds. This leads to a total principal amount of $18,049 billion, which is by far the biggest debt market. Next comes the Eurozone market, with a size of $7,100 billion, and the Japanese market, with $5,915 billion.

The domestic bond market can be further decomposed into the categories repre- senting the public and private bond markets:

**Government bonds**, issued by central governments, or also called

**bonds** (e.g., by the United States or Argentina)

**sovereign**

**Government agency and guaranteed bonds**, issued by agencies or guaranteed by the central government, (e.g., by Fannie Mae, a U.S. government agency)

**State and local bonds**, issued by local governments, other than the central gov- ernment, also known as **municipal bonds** (e.g., by the state or city of New York) Bonds issued by private **ﬁnancial institutions**, including banks, insurance compa- nies, or issuers of asset-backed securities (e.g., by Citibank in the U.S. market) **Corporate bonds**, issued by private nonﬁnancial corporations, including industri- als and utilities (e.g., by IBM in the U.S. market)

As Table 7-1 shows, the public sector accounts for more than half of the debt mar- kets. This sector includes sovereign debt issued by emerging countries in their own currencies, e.g. Mexican peso-denominated debt issued by the Mexican government. Few of these markets have long-term issues, because of their history of high inﬂation, which renders long-term bonds very risky. In Mexico, for instance, the market consists mainly of **Cetes**, which are peso-denominated, short-term Treasury Bills.

The emerging market sector also includes dollar-denominated debt, such as **Brady bonds**, which are sovereign bonds issued in exchange for bank loans, and the **Tese- bonos**, which are dollar-denominated bills issued by the Mexican government. Brady bonds are hybrid securities whose principal is collateralized by U.S. Treasury zero- coupon bonds. As a result, there is no risk of default on the principal, unlike on coupon payments.

A large and growing proportion of the market consists of mortgage-backed securities. **Mortgage-backed securities** (MBSs), or mortgage **pass-throughs**, are se- curities issued in conjunction with mortgage loans, either residential or commercial. Payments on MBSs are repackaged cash ﬂows supported by mortgage payments made by property owners. MBSs can be issued by government agencies as well as by private ﬁnancial corporations. More generally, **asset-backed securities** (ABSs) are securities whose cash ﬂows are supported by assets such as credit card receivables or car loan payments.

Finally, the remainder of the market represents bonds raised by private, nonﬁnan- cial corporations. This sector, large in the United States but smaller in other countries, is growing rather quickly as corporations recognize that bond issuances are a lower- cost source of funds than bank debt. The advent of the common currency, the Euro, is also leading to a growing, more liquid and efﬁcient, corporate bond market in Europe.

###### Fixed-Income Securities

* + 1. Instrument Types

Bonds pay interest on a regular basis, semiannual for U.S. Treasury and corporate bonds, annual for others such as Eurobonds, or quarterly for others. The most com- mon types of bonds are:

**Fixed-coupon bonds**, which pay a ﬁxed percentage of the principal every period and the principal as a **balloon**, one-time, payment at maturity

**Zero-coupon bonds**, which pay no coupons but only the principal; their return is derived from price appreciation only

**Annuities**, which pay a constant amount over time which includes interest plus amortization, or gradual repayment, of the principal;

**Perpetual bonds** or **consols**, which have no set redemption date and whose value derives from interest payments only

**Floating-coupon bonds**, which pay interest equal to a reference rate plus a margin, reset on a regular basis; these are usually called **ﬂoating-rate notes** (FRN) **Structured notes**, which have more complex coupon patterns to satisfy the in- vestor’s needs

There are many variations on these themes. For instance, **step-up bonds** have coupons that start at a low rate and increase over time.

It is useful to consider ﬂoating-rate notes in more detail. Take for instance a 10- year $100 million FRN paying semiannually 6-month LIBOR in arrears.1 Here, **LIBOR** is the London Interbank Offer Rate, a benchmark short-term cost of borrowing for AA credits. Every semester, on the **reset date**, the value of 6-month LIBOR is recorded. Say LIBOR is initially at 6%. At the next coupon date, the payment will be ( 1 ) × $100 × 6% =

2

$3 million. Simultaneously, we record a new value for LIBOR, say 8%. The next payment will then increase to $4 million, and so on. At maturity, the issuer pays the last coupon plus the principal. Like a cork at the end of a ﬁshing line, the coupon payment “ﬂoats” with the current interest rate.

Among structured notes, we should mention **inverse ﬂoaters**, which have coupon payments that vary inversely with the level of interest rates. A typical formula for the coupon is *c* = 12% — LIBOR, if positive, payable semiannually. Assume the principal is $100 million. If LIBOR starts at 6%, the ﬁrst coupon will be (1ƒ2) × $100 × (12% — 6%) = $3 million. If after six months LIBOR moves to 8%, the second coupon will be (1ƒ2) × $100 × (12% — 8%) = $2 million. The coupon will go to zero if LIBOR moves above 12%. Conversely, the coupon will increase if LIBOR drops. Hence, inverse ﬂoaters do best in a falling interest rate environment.

Bonds can also be issued with option features. The most important are:

**Callable bonds**, where the issuer has the right to “call” back the bond at ﬁxed prices on ﬁxed dates, the purpose being to call back the bond when the cost of issuing new debt is lower than the current coupon paid on the bond

1 Note that the index could be deﬁned differently. The ﬂoating payment could be tied to a Treasury rate, or LIBOR with a different maturity–say 3-month LIBOR. The pricing of the FRN will depend on the index. Also, the coupon will typically be set to LIBOR plus some spread that depends on the creditworthiness of the issuer.

**Puttable bonds**, where the investor has the right to “put” the bond back to the issuer at ﬁxed prices on ﬁxed dates, the purpose being to dispose of the bond should its price deteriorate

**Convertible bonds**, where the bond can be converted into the common stock of the issuing company at a ﬁxed price on a ﬁxed date, the purpose being to partake in the good fortunes of the company (these will be covered in Chapter 9 on equities)

The key to analyzing these bonds is to identify and price the option feature. For instance, a callable bond can be decomposed into a long position in a straight bond minus a call option on the bond price. The call feature is unfavorable for investors who will demand a lower price to purchase the bond, thereby increasing its yield. Conversely, a put feature will make the bond more attractive, increasing its price and lowering its yield. Similarly, the convertible feature allows companies to issue bonds at a lower yield than otherwise.

**Example 7-1: FRM Exam 1998---- Ques:wtion 3/Capital Markets**

7-1. The price of an inverse ﬂoater

1. Increases as interest rates increase
2. Decreases as interest rates increase
3. Remains constant as interest rates change
4. Behaves like none of the above

**Example 7-2: FRM Exam 2000---- Ques:wtion 9/Capital Markets**

7-2. An investment in a callable bond can be analytically decomposed into a

1. Long position in a noncallable bond and a short position in a put option
2. Short position in a noncallable bond and a long position in a call option
3. Long position in a noncallable bond and a long position in a call option
4. Long position in a noncallable and a short position in a call option
   * 1. Methods of Quotation

Most bonds are quoted on a **clean price** basis, that is, without accounting for the accrued income from the last coupon. For U.S. bonds, this clean price is expressed as a percent of the face value of the bond with fractions in thirty-seconds, for instance 104 — 12 or 104 + 12ƒ32 for the 6.25% May 2030 Treasury bond. Transactions are expressed in number of units, e.g. $20 million face value.

Actual payments, however, must account for the accrual of interest. This is fac- tored into the **gross price**, also known as the **dirty price**, which is equal to the clean

price plus accrued interest. In the U.S. Treasury market, accrued interest (AI) is com- puted on an *actual/actual* basis:

AI = Coupon ×

Actual number of days since last coupon Actual number of days between last and next coupon

(7*.*1)

The fraction involves the actual number of days in both the numerator and denomi- nator. For instance, say the 6.25% of May 2030 paid the last coupon on November 15 and will pay the next coupon on May 15. The denominator is, counting the number of days in each month, 15 + 31 + 31 + 29 + 31 + 30 + 15 = 182. If the trade settles

on April 26, there are 15 + 31 + 31 + 29 + 31 + 26 = 163 days into the period. The accrued is computed from the $3.125 coupon as

$3 125 × 163 $2 798763

*.* = *.*

182

The total, gross price for this transaction is:

($20*,* 000*,* 000ƒ100) × [(104 + 12ƒ32) + 2*.*798763] = $21*,* 434*,* 753

Different markets have different day count conventions. A 30/360 convention, for example, considers that all months count for 30 days exactly. The computation of the accrued interest is tedious but must be performed precisely to settle the trades.

We should note that the accrued interest in the

**LIBOR**

market is based on

*actual/360*. For instance, the actual interest payment on a 6% $1 million loan over 92 days is

$1*,* 000*,* 000 × 0*.*06 ×

92

360

= $15*,* 333*.*33

Another notable pricing convention is the discount basis for Treasury Bills. These bills are quoted in terms of an annualized discount rate (DR) to the face value, deﬁned as

DR = (Face

— P)ƒFace × (360ƒ*t*) (7*.*2)

where *P* is the price and *t* is the actual number of days. The dollar price can be recov- ered from

*P* = Face × [1 — DR × (*t*ƒ360)] (7*.*3)

For instance, a bill quoted at 5.19% discount with 91 days to maturity could be pur- chased for

$100 × [1 — 5*.*19% × (91ƒ360)] = $98*.*6881*.*

This price can be transformed into a conventional yield to maturity, using

*F* ƒ*P* = (1 + *y* × *t*ƒ365) (7*.*4)

which gives 5.33% in this case. Note that the yield is greater than the discount rate because it is a rate of return based on the initial price. Because the price is lower than the face value, the yield must be greater than the discount rate.

**Example 7-3: FRM Exam 1998---- Ques:wtion 13/Capital Markets**

7-3. A U.S. Treasury bill selling for $97,569 with 100 days to maturity and a face value of $100,000 should be quoted on a bank discount basis at

a) 8.75%

b) 8.87%

c) 8.97%

d) 9.09%

###### Analysis of Fixed-Income Securities

* + 1. The NPV Approach

Fixed-income securities can be valued by, ﬁrst, laying out their cash ﬂows and, second, discounting them at the appropriate discount rate.

This approach can also be used to infer a more convenient measure of value for the bond, which is the bond’s own yield. Let us write the market value of a bond *P* as the present value of future cash ﬂows:

*P T Ct*

= Σ (1 )*t*

+ *y*

*t*=1

(7*.*5)

where:

*Ct* =the cash ﬂow (coupon or principal) in period *t*,

*t* =the number of periods (e.g., half-years) to each payment,

*T* =the number of periods to ﬁnal maturity,

*y* =the yield to maturity for this particular bond,

*P* =the price of the bond, including accrued interest.

Here, the yield is the internal rate of return that equates the NPV of the cash ﬂows to the market price of the bond. The yield is also the expected rate of return on the bond, provided all coupons are reinvested at the same rate. For a ﬁxed-rate bond with

face value *F* , the cash ﬂow *Ct* is *cF* each period, where *c* is the coupon rate, plus *F*

upon maturity. Other cash ﬂow patterns are possible.

Figure 7-1 shows the time proﬁle of the cash ﬂows *Ct* for three bonds with initial market value of $100, 10 year maturity and 6% annual interest. The ﬁgure describes a straight coupon-paying bond, an annuity, and a zero-coupon bond. As long as the cash ﬂows are predetermined, the valuation is straightforward.

**FIGURE 7-1 Time Proﬁle of Cash Flows**

Straight-coupon

120

Principal Interest

100

80

60

40

20

0

16

14

12

10

8

6

4

2

0

200

180

160

140

120

100

80

60

40

20

0

Annuity

Zero-coupon

Problems start to arise when the cash ﬂows are random or when the life of the bond could be changed due to option-like features. In this case, the standard valuation formula in Equation (7.5) fails. More precisely, the yield cannot be interpreted as a reinvestment rate. Particularly important examples are MBSs, which are detailed in a later section.

It is also important to note that we discounted all cash ﬂows at the same rate, *y*. More generally, the fair value of the bond can be assessed using the term structure of interest rates. Deﬁne *Rt* as the **spot interest rate** for maturity *t* and this risk class (i.e., same currency and credit risk). The fair value of the bond is then:

*P*ˆ *T*  *Ct*

= Σ (1 )*t*

+ *Rt*

*t*=1

(7*.*6)

To assess whether a bond is rich or cheap, we can add a ﬁxed amount *s*, called the

**static spread** to the spot rates so that the NPV equals the current price:

*P T Ct*

= Σ (1 + *Rt* + *s*)*t*

*t*=1

(7*.*7)

All else equal, a bond with a large static spread is preferable to another with a lower spread. It means the bond is cheaper, or has a higher expected rate of return.

It is often simpler to compute a **yield spread** A*y*, using yield to maturity, such that

*P T Ct*

= Σ (1 + *y* + A*y*)*t*

*t*=1

(7*.*8)

The static spread and yield spread are conceptually similar, but the former is more accurate since the term structure is not necessarily ﬂat. When the term structure is ﬂat, the two measures are identical.

Table 7-3 gives an example of a 7% coupon, 2-year bond. The term structure en- vironment, consisting of spot rates and par yields, is described on the left side. The right side lays out the present value of the cash ﬂows (PVCF). Discounting the two

cash ﬂows at the spot rates gives a fair value of *P*ˆ = $101*.*9604. In fact, the bond is

selling at a price of *P* = $101*.*5000. So, the bond is cheap.

We can convert the difference in prices to annual yields. The yield to maturity on this bond is 6.1798%, which implies a yield spread of A*y* = 6*.*1798 — 5*.*9412 = 0*.*2386%*.* Using the static spread approach, we ﬁnd that adding *s* = 0*.*2482% to the

spot rates gives the current price. The second measure is more accurate than the ﬁrst. In this case, the difference is small. This will not be the case, however, with longer maturities and irregular yield curves.

**TABLE 7-3 Bond Price and Term Structure**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Maturity (Year)  *i* | Term Structure | | 7% Bond PVCF  Discounted at | | | | |
| Spot Rate  *Ri* | Par Yield  *yi* |
| Spot  *s* = 0 | A*y* | Yield+YS  = 0*.*2386 | *s* | Spot+SS  = 0*.*2482 |
| 1 | 4.0000 | 4.0000 | 6.7308 | 6.5926 | | 6.7147 | |
| 2 | 6.0000 | 5.9412 | 95.2296 | 94.9074 | | 94.7853 | |
| Sum |  |  | 101.9604 | 101.5000 | | 101.5000 | |
| Price |  |  | 101.5000 | 101.5000 | | 101.5000 | |

Cash ﬂows with different credit risks need to be discounted with different rates. For example, the principal on Brady bonds is collateralized by U.S. Treasury securities and carries no default risk, in contrast to the coupons. As a result, it has become com- mon to separate the discounting of the principal from that of the coupons. Valuation is done in two steps. First, the principal is discounted into a present value using the appropriate Treasury yield. The present value of the principal is subtracted from the market value. Next, the coupons are discounted at what is called the **stripped yield**, which accounts for the credit risk of the issuer.

* + 1. Duration

Armed with a cash ﬂow proﬁle, we can proceed to compute duration. As we have seen in Chapter 1, **duration** is a measure of the exposure, or sensitivity, of the bond price to movements in yields. When cash ﬂows are ﬁxed, duration is measured as the weighted maturity of each payment, where the weights are proportional to the present value of the cash ﬂows. Using the same notations as in Equation (7.5), recall that **Macaulay duration** is

*D T T Ct* ƒ(1 + *y*)*t*

*t*=1

= Σ *t*

*t*=1

× *wt* = Σ *t* × Σ *Ct* ƒ(1 + *y*)*t .*

(7*.*9)

**Key concept:**

Duration can only be viewed as the weighted average time to wait for each payment when the cash ﬂows are predetermined.

More generally, duration is a measure of interest-rate exposure:

*dP* = — *D P* = —*D*×*P*

(7*.*10)

*dy* (1 + *y*)

where *D*× is **modiﬁed duration**. The second term *D*×*P*

is also known as the **dollar**

**duration**. Sometimes this sensitivity is expressed in **dollar value of a basis point**

(also known as DV01), deﬁned as

*dP*

0*.*01% = DVBP (7*.*10)

For ﬁxed cash ﬂows, duration can be computed using Equation (7.9). Otherwise, we can infer duration from an understanding of the security. Consider a ﬂoating-rate note (FRN). Just before the reset date, we know that the coupon will be set to the prevailing interest rate. The FRN is then similar to cash, or a money market instrument, which has no interest rate risk and hence is selling at par with zero duration. Just after the reset date, the investor is locked into a ﬁxed coupon over the accrual period. The FRN is then economically equivalent to a zero-coupon bond with maturity equal to the time to the next reset date.

**Key concept:**

The duration of a ﬂoating-rate note is the time to wait until the next reset period, at which time the FRN should be at par.

**Example 7-4: FRM Exam 1999---- Ques:wtion 53/Capital Markets**

7-4. Consider a 9% annual coupon 20-year bond trading at 6% with a price of

134.41. When rates rise 10bps, price reduces to 132.99, and when rates decrease by 10bps, the price goes up to 135.85. What is the modiﬁed duration of the bond?

a) 11.25

b) 10.61

c) 10.50

d) 10.73

**Example 7-5: FRM Exam 1998---- Ques:wtion 31/Capital Markets**

7-5. A 10-year zero-coupon bond is callable annually at par (its face value) starting at the beginning of year six. Assume a ﬂat yield curve of 10%. What is the bond duration?

1. 5 years
2. 7.5 years
3. 10 years
4. Cannot be determined based on the data given

*B ∂y*

d) (1+*y*) *∂B*

*B ∂y*

c) — *y ∂B*

*B ∂y*

b) 1 *∂B*

*B ∂y*

a) — 1 *∂B*

**Example 7-6: FRM Exam 1999---- Ques:wtion 91/Market Risk**

7-6. (Modiﬁed) duration of a ﬁxed-rate bond, in the case of ﬂat yield curve, can be interpreted as (where *B* is the bond price and *y* is the yield to maturity)

**Example 7-7: FRM Exam 1997---- Ques:wtion 49/Market Risk**

7-7. A money markets desk holds a ﬂoating-rate note with an eight-year maturity. The interest rate is ﬂoating at three-month LIBOR rate, reset quarterly.

The next reset is in one week. What is the approximate duration of the ﬂoating-rate note?

1. 8 years
2. 4 years
3. 3 months
4. 1 week

###### Spot and Forward Rates

In addition to the cash ﬂows, we also need detailed information on the term structure of interest rates to value ﬁxed-income securities and their derivatives. This informa- tion is provided by **spot rates**, which are zero-coupon investment rates that start at the current time. From spot rates, we can infer **forward rates**, which are rates that start at a future date. Both are essential building blocks for the pricing of bonds.

Consider for instance a one-year rate that starts in one year. This forward rate is deﬁned as *F*1*,*2 and can be inferred from the one-year and two-year spot rates, *R*1

and *R*2. The forward rate is the break-even future rate that equalizes the return on investments of different maturities. An investor has the choice to lock in a 2-year investment at the 2-year rate, or to invest for a term of one year and roll over at the 1-to-2 year forward rate.

The two portfolios will have the same payoff when the future rate *F*1*,*2 is such that

(1 + *R*2)2 = (1 + *R*1)(1 + *F*1*,*2) (7*.*12)

For instance, if *R*1 = 4*.*00% and *R*2 = 4*.*62%, we have *F*1*,*2 = 5*.*24%.

More generally, the *T* -period spot rate can be written as a geometric average of the spot and forward rates

(1 + *RT* )*T* = (1 + *R*1)(1 + *F*1*,*2)*...*(1 + *FT* —1*,T* ) (7*.*13)

where *Fi,i*+1 is the forward rate of interest prevailing now (at time *t*) over a horizon of *i* to *i* + 1. Table 7-4 displays a sequence of spot rates, forward rates, and par yields, using annual compounding. The ﬁrst three years of this sequence are displayed in Figure 7-2.

**FIGURE 7-2 Spot and Forward Rates**

Spot rates:

R3

R2

R1

Forward rates:

F2,3

F1,2

F0,1

0 1 2 3

Forward rates allow us to project future cash ﬂows that depend on future rates. The *F*1*,*2 forward rate, for example, can be taken as the market’s expectation of the sec- ond coupon payment on an FRN with annual payments and resets. We will also show later that positions in forward rates can be taken easily with derivative instruments.

**TABLE 7-4 Spot, Forward Rates and Par Yields**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Maturity (Year)  *i* | Spot Rate  *Ri* | Forward Rate  *Fi*—1*,i* | Par Yield  *yi* | Discount Function  *D*(*ti*) |
| 1 | 4.000 | 4.000 | 4.000 | 0.9615 |
| 2 | 4.618 | 5.240 | 4.604 | 0.9136 |
| 3 | 5.192 | 6.350 | 5.153 | 0.8591 |
| 4 | 5.716 | 7.303 | 5.640 | 0.8006 |
| 5 | 6.112 | 7.712 | 6.000 | 0.7433 |
| 6 | 6.396 | 7.830 | 6.254 | 0.6893 |
| 7 | 6.621 | 7.980 | 6.451 | 0.6383 |
| 8 | 6.808 | 8.130 | 6.611 | 0.5903 |
| 9 | 6.970 | 8.270 | 6.745 | 0.5452 |
| 10 | 7.112 | 8.400 | 6.860 | 0.5030 |

Forward rates have to be positive, otherwise there would be an arbitrage opportu- nity. We abstract from transaction costs and assume we can invest and borrow at the same rate. For instance, *R*1 = 11*.*00% and *R*2 = 4*.*62% gives *F*1*,*2 = —1*.*4%. This means that (1 + *R*1) = 1*.*11 is greater than (1 + *R*2)2 = 1*.*094534. To take advantage of this discrepancy, we could borrow $1 million for two years and invest it for one year. After the ﬁrst year, the proceeds are kept in cash, or under the proverbial mattress, for the second period. The investment gives $1,110,000 and we have to pay back $1,094,534 only. This would create a proﬁt of $15,466 out of thin air, which is highly unlikely in practice. Interest rates must be positive for the same reason.

The forward rate can be interpreted as a measure of the slope of the term structure. We can, for instance, expand both sides of Equation (7.12). After neglecting cross- product terms, we have

*F*1*,*2 = *R*2 +(*R*2 — *R*1) (7*.*14)

Thus, with an upward sloping term structure, *R*2 is above *R*1, and *F*1*,*2 will also be above *R*2.

We can also show that in this situation, the spot rate curve is above the par yield curve. Consider a bond with 2 payments. The 2-year par yield *y*2 is implicitly deﬁned from:

*P* = *cF* + (*cF* + *F* )

*cF* (*cF* + *F* )

= +

(1 + *y*2) (1 + *y*2)2

(1 + *R*1) (1 + *R*2)2

where *P* is set to par *P* = *F* . The par yield can be viewed as a weighted average of spot rates. In an upward-sloping environment, par yield curves involve coupons that are discounted at shorter and thus lower rates than the ﬁnal payment. As a result, the par yield curve lies below the spot rate curve.

For a formal proof, consider a 2-period par bond with a face value of $1 and coupon of *y*2. We can write the price of this bond as

1 = *y*2ƒ(1 + *R*1) +(1 + *y*2)ƒ(1 + *R*2)2 (1 + *R*2)2 = *y*2(1 + *R*2)2ƒ(1 + *R*1) +(1 + *y*2) (1 + *R*2)2 = *y*2(1 + *F*1*,*2) +(1 + *y*2)

2*R*2 + *R*2 = *y*2(1 + *F*1*,*2) + *y*2

2

*y*2 = *R*2(2 + *R*2)ƒ(2 + *F*1*,*2)

In an upward-sloping environment, *F*1*,*2 > *R*2 and thus *y*2 < *R*2.

When the spot rate curve is ﬂat, the spot curve is identical to the par yield curve and to the forward curve. In general, the curves differ. Figure 7-3a displays the case of an upward sloping term structure. It shows the yield curve is below the spot curve while the forward curve is above the spot curve. With a downward sloping term structure, as shown in Figure 7-3b, the yield curve is above the spot curve, which is above the forward curve.

**Example 7-8: FRM Exam 1998---- Ques:wtion 39/Capital Markets**

7-8. Which of the following statements about yield curve arbitrage is *true*?

1. No-arbitrage conditions require that the zero-coupon yield curve is either upward sloping or downward sloping.
2. It is a violation of the no-arbitrage condition if the one-year interest rate is 10% or more, higher than the 10-year rate.
3. As long as all discount factors are less than one but greater than zero, the curve is arbitrage free.
4. The no-arbitrage condition requires all forward rates be nonnegative.

**FIGURE 7-3a Upward-Sloping Term Structure**

Interest rate

9

Forward curve

Spot curve

Par yield curve

8

7

6

5

4

3

0 1 2 3 4 5 6 7 8 9 10

Maturity (Year)

**Example 7-9: FRM Exam 1997---- Ques:wtion 1/Quantitative Techniques**

7-9. Suppose a risk manager has made the mistake of valuing a zero-coupon bond using a swap (par) rate rather than a zero-coupon rate. Assume the par curve is upward sloping. The risk manager is therefore

1. Indifferent to the rate used
2. Over-estimating the value of the bond
3. Under-estimating the value of the bond
4. Lacking sufﬁcient information

**Example 7-10: FRM Exam 1999---- Ques:wtion 1/Quant. Analysis**

7-10. Suppose that the yield curve is upward sloping. Which of the following statements is *true*?

1. The forward rate yield curve is above the zero-coupon yield curve, which is above the coupon-bearing bond yield curve.
2. The forward rate yield curve is above the coupon-bearing bond yield curve, which is above the zero-coupon yield curve.
3. The coupon-bearing bond yield curve is above the zero-coupon yield curve, which is above the forward rate yield curve.
4. The coupon-bearing bond yield curve is above the forward rate yield curve, which is above the zero-coupon yield curve.

**FIGURE 7-3b Downward-Sloping Term Structure**

Interest rate

11

Par yield curve

Spot curve

Forward curve

10

9

8

0 1 2 3 4 5 6 7 8 9 10

Maturity (Year)

###### Mortgage-Backed Securities

7.5.1 Description

Mortgage-backed securities represent claims on repackaged mortgage loans. Their ba- sic cash-ﬂow patterns start from an annuity, where the homeowner makes a monthly ﬁxed payment that covers principal and interest.

Whereas mortgage loans are subject to credit risk, due to the possibility of default by the homeowner, most traded securities have third-party guarantees against credit risk. For instance, MBSs issued by Fannie Mae, an agency that is sponsored by the

U.S. government, carry a guarantee of full interest and principal payment, even if the original borrower defaults.

Even so, MBSs are complex securities due to the uncertainty in their cash ﬂows. Con- sider the traditional ﬁxed-rate mortgage. Homeowners have the possibility of making early payments of principal. This represents a long position in an option. In some cases, these prepayments are random, for instance when the homeowner sells the home due to changing job or family conditions. In other cases, these prepayments are more pre- dictable. When interest rates fall, prepayments increase as homeowners can reﬁnance at a lower cost. This is similar to the rational early exercise of American call options.

Generally, these factors affect reﬁnancing patterns:

*Age of the loan*: Prepayment rates are generally low just after the mortgage loan has been issued and gradually increase over time until they reach a stable, or “seasoned,” level. This effect is known as **seasoning**.

*Spread between the mortgage rate and current rates*: Like a callable bond, there is a greater beneﬁt to reﬁnancing if it achieves a signiﬁcant cost saving.

*Reﬁnancing incentives*: The smaller the costs of reﬁnancing, the more likely home- owners will reﬁnance often.

*Previous path of interest rates*: Reﬁnancing is more likely to occur if rates have been high in the past but recently dropped. In this scenario, past prepayments have been low but should rise sharply. In contrast, if rates are low but have been so for a while, most of the principal will already have been prepaid. This path dependence is usually referred to as **burnout**.

*Level of mortgage rates*: Lower rates increase affordability and turnover. *Economic activity*: An economic environment where more workers change job lo- cation creates greater job turnover, which is more likely to lead to prepayments. *Seasonal effects*: There is typically more home-buying in the Spring, leading to increased prepayments in early Fall.

The prepayment rate is summarized into what is called the **conditional prepay- ment rate (CPR)**, which is expressed in annual terms. This number can be translated into a monthly number, known as the **single monthly mortality (SMM) Rate** using the adjustment:

(1 — SMM)12 = (1 — CPR) (7*.*15)

For instance, if CPR = 6% annually, the monthly proportion of principal paid early will be SMM = 1 — (1 — 0*.*06)1ƒ12 = 0*.*005143, or 0.514% monthly. For a loan with a be- ginning monthly balance (BMB) of BMB = $50,525 and a scheduled principal payment of SP = $67, the prepayment will be 0*.*005143 × ($50*,*525 — $67) = $260.

To price the MBS, the portfolio manager should describe the schedule of prepay- ments during the remaining life of the bond. This depends on many factors, including the age of the loan.

Prepayments can be described using an industry standard, known as the **Public Securities Association (PSA)** prepayment model. The PSA model assumes a CPR of

0.2% for the ﬁrst month, going up by 0.2% per month for the next 30 months, until 6% thereafter. Formally, this is:

CPR = Min[6% × (*t*ƒ30)*,* 6%] (7*.*16)

This pattern is described in Figure 7-4 as the 100% PSA speed. By convention, prepay- ment patterns are expressed as a percentage of the PSA speed, for example 165% for a faster pattern and 70% PSA for a slower pattern.

**Example:**

Computing the CPR Consider an MBS issued 20 months ago with a speed of 150% PSA. What are the CPR and SMM?

The PSA speed is Min[6% × (20ƒ30)*,* 6%] = 0*.*04*.* Applying the 150 factor, we have

CPR = 150% × 0*.*04 = 0*.*06. This implies SMM = 0*.*514%.

**FIGURE 7-4 Prepayment Pattern**

Annual CPR percentage

10

165% PSA

100% PSA

70% PSA

9

8

7

6

5

4

3

2

1

0

0 10 20 30 40 50

Mortgage age (months)

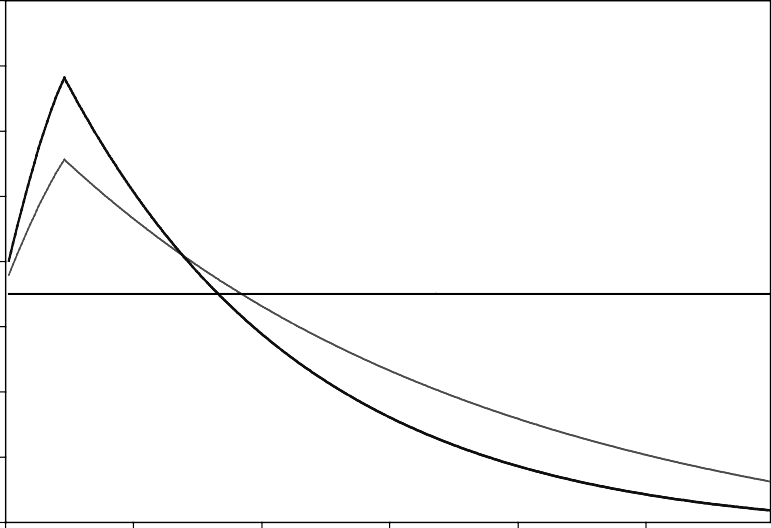
The next step is to project cash ﬂows based on the prepayment speed pattern. Figure 7-5 displays cash-ﬂow patterns for a 30-year MBS with a face amount of $100 million, 7.5% interest rate, and three months into its life. The horizontal, “0% PSA” line, describes the annuity pattern without any prepayment. The “100% PSA” line describes

an increasing pattern of cash ﬂows, peaking in 27 months and decreasing thereafter. This point corresponds to the stabilization of the CPR at 6%. This pattern is more marked for the “165% PSA” line, which assumes a faster prepayment speed.

Early prepayments create less payments later, which explains why the 100% PSA line is initially greater than the 0% line, then lower later as the principal has been paid off more quickly.

**FIGURE 7-5 Cash Flows on MBS for Various PSA**

Cash flow ($ million)

1.6

1.4

1.2

1.0

0.8 0% PSA

0.6

0.4

0.2

100%PSA

165%PSA

0

0 60 120 180 240 300

Months to maturity

**Example 7-11: FRM Exam 1999---- Ques:wtion 51/Capital Markets**

7-11. Suppose the annual prepayment rate CPR for a mortgage-backed security is 6%. What is the corresponding single-monthly mortality rate SMM?

a) 0.514%

b) 0.334%

c) 0.5%

d) 1.355%

**Example 7-12: FRM Exam 1998---- Ques:wtion 14/Capital Markets**

7-12. In analyzing the monthly prepayment risk of Mortgage-backed securities, an annual prepayment rate (CPR) is converted into a monthly prepayment rate (SMM). Which of the following formulas should be used for the conversion?

a) SMM = (1 — CPR)1ƒ12

b) SMM = 1 — (1 — CPR)1ƒ12

c) SMM = 1 — (CPR)1ƒ12

d) SMM = 1 +(1 — CPR)1ƒ12

**Example 7-13: FRM Exam 1999---- Ques:wtion 87/Market Risk**

7-13. A CMO bond class with a duration of 50 means that

1. It has a discounted cash ﬂow weighted average life of 50 years.
2. For a 100 bp change in yield, the bond’s price will change by roughly 50%. c) For a 1 bp change in yield, the bond’s price will change by roughly 5%.

d) None of the above is correct.

**Example 7-14: FRM Exam 1998---- Ques:wtion 18/Capital Markets**

7-14. Which of the following risks are common to both mortgage-backed securities and emerging market Brady bonds?

1. Interest rate risk
2. Prepayment risk
3. Default risk
4. Political risk
5. I only
6. II and III only
7. I and III only
8. III and IV only

7.5.2 Prepayment Risk

Like other bonds, mortgage-backed securities are subject to market risk, due to ﬂuc- tuations in interest rates. They are also, however, subject to **prepayment risk**, which is the risk that the principal will be repaid early.

Consider for instance an 8% MBS, which is illustrated in Figure 7-6. If rates drop to 6%, homeowners will rationally prepay early to reﬁnance the loan. Because the av- erage life of the loan is shortened, this is called **contraction risk**. Conversely, if rates increase to 10%, homeowners will be less likely to reﬁnance early, and prepayments

will slow down. Because the average life of the loan is extended, this is called **exten- sion risk**.

As shown in Figure 7-6, these features create “negative convexity”, which reﬂects the fact that the investor in an MBS is short an option. At point B, interest rates are very high and there is little likelihood that the homeowner will reﬁnance early. The option is nearly worthless and the MBS behaves like a regular bond, with positive convexity. At point A, the option pattern starts to affect the value of the MBS. Shorting an option creates negative gamma, or convexity.

**FIGURE 7-6 Negative Convexity of MBSs**

Market price

140

A

Positive convexity

B

Negative convexity

120

100

80

60

40

20

0

5 6 7 8 9 10 11 12

Market yield

This changing cash-ﬂow pattern makes standard duration measures unreliable. Instead, sensitivity measures are computed using **effective duration** and **effective convexity**, as explained in Chapter 1. The measures are based on the estimated price of the MBS for three yield values, making suitable assumptions about how changes in rates should affect prepayments.

Table 7-5 shows an example. In each case, we consider an upmove and downmove of 25bp. In the ﬁrst, “unchanged” panel, the PSA speed is assumed to be constant at 165 PSA. In the second, “changed” panel, we assume a higher PSA speed if rates drop and lower speed if rates increase. When rates drop, the MBS value goes up but not as

much as if the prepayment speed had not changed, which reﬂects contraction risk. When rates increase, the MBS value drops by more than if the prepayment speed had not changed, which reﬂects extension risk.

**TABLE 7-5 Computing Effective Duration and Convexity**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Initial | Unchanged PSA | | Changed PSA | |
| Yield | 7.50% | +25bp | —25bp | +25bp | —25bp |
| PSA |  | 165PSA | 165PSA | 150PSA | 200PSA |
| Price | 100.125 | 98.75 | 101.50 | 98.7188 | 101.3438 |
| Duration |  | 5.49y | | 5.24y | |
| Convexity | 0 | | —299 | |

As we have seen in Chapter 1, **effective duration** is measured as

*DE* = *P* (*y*0 — A*y*) — *P* (*y*0 + A*y*) (7*.*17) (2*P*0A*y*)

**Effective convexity** is measured as

= —[ ]

*CE P* (*y*0 — A*y*) — *P*0 *P*0 — *P* (*y*0 + A*y*) (*P*0A*y*) (*P*0A*y*)

ƒA*y*

(7*.*18)

In the ﬁrst, “unchanged” panel, the effective duration is 5.49 years and convexity close to zero. In the second, “changed” panel, the effective duration is 5.24 years and convexity is negative, as expected, and quite large.

**Key concept:**

Mortgage-backed securities have negative convexity, which reﬂects the short position in an option granted to the homeowner to repay early. This creates extension risk when rates increase or contraction risk when rates fall.

The option feature in MBSs increases their yield. To ascertain whether the securi- ties represent good value, portfolio managers need to model the option component. The approach most commonly used is the **option-adjusted spread** (OAS).

Starting from the **static spread**, the OAS method involves running simulations of various interest rate scenarios and prepayments to establish the option cost. The OAS is then

OAS = Static spread — Option cost (7*.*19)

which represents the net richness or cheapness of the MBS. Within the same risk class, a security trading at a high OAS is preferable to others.

The OAS is more stable over time than the spread, because the latter is affected by the option component. This explains why during market rallies (i.e., when long-term Treasury yields fall) yield spreads on current coupon mortgages often widen. These mortgages are more likely to be prepaid early, which makes them less attractive. Their option cost increases, pushing up the yield spread.

**Example 7-15: FRM Exam 1999---- Ques:wtion 44/Capital Markets**

7-15. The following are reasons that a prepayment model will not accurately predict future mortgage prepayments. Which of these will have the greatest effect on the convexity of mortgage pass-throughs?

1. Reﬁnancing incentive
2. Seasoning
3. Reﬁnancing burnout
4. Seasonality

**Example 7-16: FRM Exam 1999---- Ques:wtion 40/Capital Markets**

7-16. Which attribute of a bond is *not* a reason for using effective duration instead of modiﬁed duration?

1. Its life may be uncertain.
2. Its cash ﬂow may be uncertain.
3. Its price volatility tends to decline as maturity approaches.
4. It may include changes in adjustable rate coupons with caps or ﬂoors.

**Example 7-17: FRM Exam 2001---- Ques:wtion 95**

7-17. The option-adjusted duration of a callable bond will be close to the duration of a similar non-callable bond when the

1. Bond trades above the call price.
2. Bond has a high volatility.
3. Bond trades much lower than the call price.
4. Bond trades above parity.

7.5.3 Financial Engineering and CMOs

The MBS market has grown enormously in the last twenty years in the United States and is growing fast in other markets. MBSs allow capital to ﬂow from investors to borrowers, principally homeowners, in an efﬁcient fashion.

One major drawback of MBSs, however, is their negative convexity. This makes it difﬁcult for investors, such as pension funds, to invest in MBSs because the life of these instruments is uncertain, making it more difﬁcult to match the duration of pension assets to the horizon of pension liabilities.

In response, the ﬁnance industry has developed new classes of securities based on MBSs with more appealing characteristics. These are the **collateralized mortgage obligations (CMOs)**, which are new securities that redirect the cash ﬂows of an MBS pool to various segments.

Figure 7-7 illustrates the process. The cash ﬂows from the MBS pool go into a **special-purpose vehicle (SPV)**, which is a legal entity that issues different claims, or **tranches** with various characteristics, like slices in a pie. These are structured so that the cash ﬂow from the ﬁrst tranche, for instance, is more predictable than the original cash ﬂows. The uncertainty is then pushed into the other tranches.

Starting from an MBS pool, ﬁnancial engineering creates securities that are better tailored to investors’ needs. It is important to realize, however, that the cash ﬂows and risks are fully preserved. They are only redistributed across tranches. Whatever transformation is brought about, the resulting package must obey basic laws of con- servation for the underlying securities and package of resulting securities. We must have the same cash ﬂows at each point in time, except for fees paid to the issuer. As a result, we must have

1. The same market value
2. The same risk proﬁle

As Lavoisier, the French chemist who was executed during the French revolution said,

*Rien ne se perd, rien ne se cr´ee (nothing is lost, nothing is created).*

In particular, the weighted duration and convexity of the portfolio of tranches must add up to the original duration and convexity. If Tranche A has less convexity than the underlying securities, the other tranches must have more convexity.

Similar structures apply to **collateralized bond obligations** (CBOs), **collateralized loan obligations** (CLOs), **collateralized debt obligations** (CDOs), which are a set of tradable bonds backed by bonds, loans, or debt (bonds and loans), respectively. These structures rearrange credit risk and will be explained in more detail in a later chapter. As an example of a two-tranche structure, consider a claim on a regular 5-year, 6% coupon $100 million note. This can be split up into a ﬂoater, that pays LIBOR on a

**FIGURE 7-7 Creating CMO Tranches**

**Mortgage loans**

**Pass-Through: Pool of Mortgage**

**Obligations**

Special Purpose Vehicle

notional of $50 million, and an inverse ﬂoater, that pays 12% — LIBOR on a notional of $50 million. The coupon on the inverse ﬂoater cannot go below zero: Coupon =

|  |  |
| --- | --- |
| **Tranche A** | **Cash Flow** |
|  |
| **Tranche B** |
|  |
| **Tranche C** |
|  |
| **Tranche Z** |
|  |

Max(12%—LIBOR*,* 0). This imposes another condition on the ﬂoater Coupon = Min(LIBOR*,* 12%).

We verify that the cash ﬂows exactly add up to the original. For each coupon pay- ment, we have, in millions

$50 × LIBOR +$50 × (12% — LIBOR) = $100 × 6% = $6*.*

At maturity, the total payments of twice $50 million add up to $100 million.

We can also decompose the risk of the original structure into that of the two com- ponents. Assume a ﬂat term structure for the original note. Say the duration of the original 5-year note is *D* = 4*.*5 years. The portfolio dollar duration is:

$50*,* 000*,* 000 × *DF* +$50*,* 000*,* 000 × *D*IF = $100*,* 000*,* 000 × *D*

Just before a reset, the duration of the ﬂoater is close to zero *DF* = 0. Hence, the duration of the inverse ﬂoater must be *D*IF = ($100*,* 000*,* 000ƒ$50*,* 000*,* 000) × *D* = 2 × *D*, or twice that of the original note. Note that the duration is much greater than the maturity of the note. This illustrates the point that duration is an interest rate sensitivity measure. When cash ﬂows are uncertain, duration is not necessarily related to maturity. Intuitively, the ﬁrst tranche, the ﬂoater, has zero risk so that all of the

risk must be absorbed into the second tranche, which must have a duration of 9 years. The total risk of the portfolio is conserved.

This analysis can be easily extended to inverse ﬂoaters with greater leverage. Sup- pose the coupon the coupon is tied to twice LIBOR, for example 18% — 2 × LIBOR. The principal must be allocated in the amount *x*, in millions, for the ﬂoater and 100 — *x* for the inverse ﬂoater so that the coupon payment is preserved. We set

*x* × *LIBOR* +(100 — *x*) × (18% — 2 × *LIBOR*) = $6

[*x* — (100 — *x*)2] × *LIBOR* +(100 — *x*) × 18% = $6

This can only be satisﬁed if 3*x* — 200 = 0, or if *x* = $66*.*67 million. Thus, two-thirds of the notional must be allocated to the ﬂoater, and one-third to the inverse ﬂoater. The inverse ﬂoater now has three times the duration of the original note.

**Key concept:**

Collateralized mortgage obligations (CMOs) rearrange the total cash ﬂows, total value, and total risk of the underlying securities. At all times, the total cash ﬂows, value, and risk of the tranches must equal those of the collateral. If some tranches are less risky than the collateral, others must be more risky.

When the collateral is a mortgage-backed security, CMOs can be deﬁned by priori- tizing the payment of principal into different tranches. This is deﬁned as **sequential- pay tranches**. Tranche A, for instance, will receive the principal payment on the whole underlying mortgages ﬁrst. This creates more certainty in the cash ﬂows accruing to Tranche A, which makes it more appealing to some investors. Of course, this is to the detriment of others. After principal payments to Tranche A are exhausted, Tranche B then receives all principal payments on the underlying MBS. And so on for other tranches.

Another popular construction is the IO/PO structure. An **interest-only (IO)** tranche receives only the interest payments on the underlying MBS. The **principal-only (PO)** tranche then receives only the principal payments. As before, the market value of the IO and PO must exactly add to that of the MBS. Figure 7-8 describes the price behavior of the IO and PO. Note that the vertical addition of the two components always equals the value of the MBS.

**FIGURE 7-8 Creating an IO and PO from an MBS**

Market price

140

Pass-Through

Interest-Only (IO)

Principal-Only (PO)

120

100

80

60

40

20

0

5 6 7 8 9 10 11 12

Market yield

To analyze the PO, it is useful to note that the sum of all principal payments is constant (because we have no default risk). Only the timing is uncertain. In contrast, the sum of all interest payments depends on the timing of principal payments. Later principal payments create greater total interest payments.

If interest rates fall, principal payments will come early, which reﬂects contraction risk. Because the principal is paid earlier and the discount rate decreases, the PO should appreciate sharply in value. On the other hand, the faster prepayments mean less interest payments over the life of the MBS, which is unfavorable to the IO. the IO should depreciate.

Conversely, if interest rates rise, slower prepayments will slow down, which re- ﬂects extension risk. Because the principal is paid later and the discount rate in- creases, the PO should lose value. On the other hand, the slower prepayments mean more interest payments over the life of the MBS, which is favorable to the IO. The IO appreciates in value, up to the point where the higher discount rate effect dominates. Thus, IOs are bullish securities with negative duration, as shown in Figure 7-8.

**Example 7-18: FRM Exam 2000---- Ques:wtion 13/Capital Markets**

7-18. A CLO is generally

1. A set of loans that can be traded individually in the market
2. A pass-through
3. A set of bonds backed by a loan portfolio
4. None of the above

**Example 7-19: FRM Exam 2000---- Ques:wtion 121/Quant. Analysis**

7-19. Which one of the following long positions is more exposed to an increase in interest rates?

1. A Treasury Bill
2. 10-year ﬁxed-coupon bond
3. 10-year ﬂoater
4. 10-year reverse ﬂoater

**Example 7-20: FRM Exam 1998---- Ques:wtion 32/Capital Markets**

7-20. A 10-year reverse ﬂoater pays a semiannual coupon of 8% minus 6-month LIBOR. Assume the yield curve is 8% ﬂat, the current 10-year note has a duration of 7 years, and the interest rate on the note was just reset. What is the duration of the note?

1. 6 months
2. Shorter than 7 years
3. Longer than 7 years
4. 7 years

**Example 7-21: FRM Exam 1999---- Ques:wtion 79/Market Risk**

7-21. Suppose that the coupon and the modiﬁed duration of a 10-year bond priced to par are 6.0% and 7.5, respectively. What is the approximate modiﬁed duration of a 10-year inverse ﬂoater priced to par with a coupon of

18% — 2 × LIBOR?

a) 7.5

b) 15.0

c) 22.5

d) 0.0

**Example 7-22: FRM Exam 2000---- Ques:wtion 3/Capital Markets**

7-22. How would you describe the typical price behavior of a low premium mortgage pass-through security?

1. It is similar to a U.S. Treasury bond.
2. It is similar to a plain vanilla corporate bond.
3. When interest rates fall, its price increase would exceed that of a comparable duration U.S. Treasury bond.
4. When interest rates fall, its price increase would lag that of a comparable duration U.S. Treasury bond.

###### Answers to Chapter Examples

**Example 7-1: FRM Exam 1998---- Ques:wtion 3/Capital Markets**

b) As interest rates increase, the coupon decreases. In addition, the discount factor increases. Hence, the value of the note must decrease even more than a regular ﬁxed- coupon bond.

**Example 7-2: FRM Exam 2000---- Ques:wtion 9/Capital Markets**

d) With a callable bond the issuer has the option to call the bond early if its price would otherwise go up. Hence, the investor is short an option. A long position in a callable bond is equivalent to a long position in a noncallable bond plus a short position in a call option.

**Example 7-3: FRM Exam 1998---- Ques:wtion 13/Capital Markets**

a) DR = (Face — Price)ƒFace × (360ƒ*t*) = ($100*,*000 — $97*,*569)ƒ$100*,*000 × (360ƒ100) =

8*.*75%*.* Note that the yield is 9.09%, which is higher.

**Example 7-4: FRM Exam 1999---- Ques:wtion 53/Capital Markets**

b) Using Equation (7.8), we have *D*× = —(*dP* ƒ*P* )ƒ*dy* = [(135*.*85 — 132*.*99)ƒ134*.*41]ƒ

[0*.*001 × 2] = 10*.*63*.* This is also a measure of effective duration.

**Example 7-5: FRM Exam 1998---- Ques:wtion 31/Capital Markets**

c) Because this is a zero-coupon bond, it will always trade below par, and the call should never be exercised. Hence its duration is the maturity, 10 years.

**Example 7-6: FRM Exam 1999---- Ques:wtion 91/Market Risk**

a) By Equation (7.8).

**Example 7-7: FRM Exam 1997---- Ques:wtion 49/Market Risk**

d) Duration is not related to maturity when coupons are not ﬁxed over the life of the investment. We know that at the next reset, the coupon on the FRN will be set at the prevailing rate. Hence, the market value of the note will be equal to par at that time. The duration or price risk is only related to the time to the next reset, which is 1 week here.

**Example 7-8: FRM Exam 1998---- Ques:wtion 39/Capital Markets**

d) Discount factors need to be below one, as interest rates need to be positive, but in addition forward rates also need to be positive.

**Example 7-9: FRM Exam 1997---- Ques:wtion 1/Quantitative Techniques**

b) If the par curve is rising, it must be below the spot curve. As a result, the discounting will use rates that are too low, thereby overestimating the bond value.

**Example 7-10: FRM Exam 1999---- Ques:wtion 1/Quant. Analysis**

1. See Figures 7-3a an 7-3b. The coupon yield curve is an average of the spot, zero- coupon curve, hence has to lie below the spot curve when it is upward-sloping. The forward curve can be interpreted as the spot curve plus the slope of the spot curve. If the latter is upward sloping, the forward curve has to be above the spot curve.

**Example 7-11: FRM Exam 1999---- Ques:wtion 51/Capital Markets**

a) Using (1 — 6%) = (1 — SMM)12, we ﬁnd SMM = 0.51%.

**Example 7-12: FRM Exam 1998---- Ques:wtion 14/Capital Markets**

b) As (1 — SMM)12 = (1 — CPR).

**Example 7-13: FRM Exam 1999---- Ques:wtion 87/Market Risk**

1. Discounted cash ﬂows are not useful for CMOs because they are uncertain. So, du- ration is a measure of interest rate sensitivity. We have (*dP* ƒ*P* ) = *D*×*dy* = 50 × 1% = 50%*.*

**Example 7-14: FRM Exam 1998---- Ques:wtion 18/Capital Markets**

1. MBSs are subject to I, II, III (either homeowner or agency default). Brady bonds are subject to I, III, IV. Neither is exposed to currency risk.

**Example 7-15: FRM Exam 1999---- Ques:wtion 44/Capital Markets**

a) The question is which factor has the greatest effect on the interest rate convexity, or increases the prepayment rate when rates fall . Seasoning and seasonality are not re- lated to interest rates. Burnout lowers the prepayment rate. So, reﬁnancing incentives is the remaining factor that affects most the option feature.

**Example 7-16: FRM Exam 1999---- Ques:wtion 40/Capital Markets**

c) Effective convexity is useful when the cash ﬂows are uncertain. All attributes are reasons for using effective convexity, except that the price risk decreases as maturity gets close. This holds for a regular coupon-paying bond anyway.

**Example 7-17: FRM Exam 2001---- Ques:wtion 95**

c) This question is applicable to MBSs as well as callable bonds. From Figure 7-6, we see that the callable bond behaves like a straight bond when market yields are high, or when the bond price is low. So, (c) is correct and (a) and (d) must be wrong.

**Example 7-18: FRM Exam 2000---- Ques:wtion 13/Capital Markets**

1. Like a CMO, a CLO represents a set of tradable securities backed by some collateral, in this case a loan portfolio.

**Example 7-19: FRM Exam 2000---- Ques:wtion 121/Quant. Analysis**

1. Risk is measured by duration. Treasury bills and ﬂoaters have very small duration. A 10-year ﬁxed-rate note will have a duration of perhaps 8 years. In contrast, an inverse (or reverse) ﬂoater has twice the duration.

**Example 7-20: FRM Exam 1998---- Ques:wtion 32/Capital Markets**

c) The duration is normally about 14 years. Note that if the current coupon is zero, the inverse ﬂoater behaves like a zero-coupon bond with a duration of 10 years.

**Example 7-21: FRM Exam 1999---- Ques:wtion 79/Market Risk**

1. Following the same reasoning as above, we must divide the ﬁxed-rate bonds into 2/3 FRN and 1/3 inverse ﬂoater. This will ensure that the inverse ﬂoater payment is related to twice LIBOR. As a result, the duration of the inverse ﬂoater must be 3 times that of the bond.

**Example 7-22: FRM Exam 2000---- Ques:wtion 3/Capital Markets**

1. MBSs are unlike regular bonds, Treasuries, or corporates, because of their nega- tive convexity. When rates fall, homeowners prepay early, which means that the price appreciation is less than that of comparable duration regular bonds.